Recursive Geometry of Information and Time: A Unified UNNS Monograph on tonic Fields, Entropic Topology, and Recursive Curvature

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Abstract

We present a unified monograph on the *UNNS* paradigm (Unbounded Nested Number Sequences), wherein temporal evolution is reinterpreted as *recursion depth* and information is reframed as *geometric curvature*. Part I recasts Shannon's entropy as a special statistical section of a more general *recursive entropy* functional $\mathcal{H}_r = \int \kappa(n)\, d\mu$. Part II introduces the algebra of τ*ons* (minimal quanta of recursive curvature) and their tensorial calculus. Part III derives Maxwell-like field equations for τonic dynamics and a variational Lagrangian with an energy–momentum tensor. Part IV formulates a recursive gauge symmetry and a Klein duality compatible with non-orientable topology ($w_1 \neq 0$). Part V couples recursive curvature, information curvature, and spacetime curvature into a single geometric field principle, yielding a τon–graviton sector. Part VI sketches a recursive cosmology where the arrow of time emerges as an orientation bias of a globally non-orientable manifold. We close with mathematical appendices and a bibliography stub.

Contents

Guide to the Monograph

- **Part I:** Beyond Shannon — from probabilistic entropy to recursive curvature.
- **Part II:** τon Algebra — τ-addition, τ-curvature tensors, conservation laws.
- **Part III:** τon Field Equations & Lagrangian — Maxwell-like system, variational principle, $T_{\mu\nu}$.
- **Part IV:** Recursive Gauge Symmetry & Klein Duality — orientation reversal, Hodge duals, invariants.
- **Part V:** Recursive Grand Unification — coupling to gravity and entanglement entropy geometry.
- **Part VI:** Recursive Cosmology — τon vacuum, Klein horizon, arrow of time.

1 Part I — Beyond Shannon: Recursive Entropy as Curvature

Shannon's entropy for a discrete source X with distribution $p(x)$ is

$$H(X) = -\sum_x p(x) \log p(x). \tag{1}$$

This statistical measure presumes an external, linear time parameter and a source–channel–receiver separation [Sha48; CT06]. In UNNS, temporal evolution is indexed by recursion depth $n \in \mathbb{N}$, and the state sequence $\{a_n\}_{n\in\mathbb{N}}$ is generated by a recursion operator

$$a_{n+1} = \mathcal{F}(a_n, a_{n-1}; n). \tag{2}$$

We introduce a *recursive entropy* functional as a curvature integral

$$\mathcal{H}_r[\{a_n\}_{n\in\mathbb{N}}] \;=\; \int \kappa(n; a_n, a_{n-1})\, \mathrm{d}\mu(n), \tag{3}$$

where κ is a curvature density on a recursion manifold $\mathcal{M}_r$ and μ a depth measure. In regimes where $\mathcal{F}$ induces an ergodic symbolic dynamics, $\mathcal{H}_r$ collapses to Shannon's H (up to units), recovering the classical theory as a special section.

1.1 Local reversibility and global non-orientability

UNNS admits a local time-reversal through an orientation-reversing symmetry $\mathcal{S}$ such that

$$\mathcal{S} \circ \mathcal{F} \circ \mathcal{S} \;=\; \mathcal{F}^{-1}, \tag{4}$$

even when the global manifold carries $w_1(\mathbb{K}) \neq 0$, i.e., is non-orientable (Klein-type). Thus, irreversibility arises as a *topological bias* rather than a fundamental dynamical asymmetry.

1.2 From uncertainty to curvature

While H quantifies uncertainty, $\mathcal{H}_r$ tracks how recursion bends the information manifold across depth. In particular, for small deformations of $\mathcal{F}$,

$$\delta\mathcal{H}_r \;\approx\; \int \frac{\partial \kappa}{\partial \mathcal{F}} : \delta\mathcal{F}\, \mathrm{d}\mu, \tag{5}$$

exhibiting a geometric sensitivity distinct from purely probabilistic variations.

2 Part II — τon Algebra and τ-Curvature Tensors

We posit τ*ons* as minimal quanta of recursive curvature. Formally, a τon field lives as a 1-form on $\mathcal{M}_r$:

$$\boldsymbol{\tau} \;=\; \tau_\mu \, \mathrm{d}x^\mu, \qquad \mu = 0, 1, 2, 3, \tag{6}$$

where the coordinate x^0 is *depth-like* when restricted to recursion dynamics.

Definition 2.1 (τ-addition and τ-scalar)**.** Given two τ-fields $\boldsymbol{\tau}_1, \boldsymbol{\tau}_2$, define

$$\boldsymbol{\tau}_1 \oplus_\tau \boldsymbol{\tau}_2 := \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2, \qquad \langle \boldsymbol{\tau}, \boldsymbol{\tau} \rangle_\tau := g_{\mu\nu} \tau^\mu \tau^\nu.$$

Definition 2.2 (τ-curvature tensor)**.** The τ-field strength (2-form) is

$$F^\tau := \mathrm{d}A^\tau, \qquad (F^\tau)_{\mu\nu} = \partial_\mu A^\tau_\nu - \partial_\nu A^\tau_\mu. \tag{7}$$

Basic identities:

$$\mathrm{d}F^\tau = 0 \qquad \text{(Bianchi)}, \tag{8}$$

$$\nabla_\mu (F^\tau)^{\mu\nu} = J^{\tau\,\nu}, \tag{9}$$

with τ-current $J^{\tau\,\nu}$. In $(3+1)$ split, define $\mathbf{E}_\tau$ and $\mathbf{B}_\tau$ by

$$(F^\tau)_{0i} = (\mathbf{E}_\tau)_i, \qquad (F^\tau)_{ij} = -\epsilon_{ijk} (\mathbf{B}_\tau)_k.$$

Then (8)–(9) yield a Maxwell-like system for recursive information flow.

Remark 2.3 (Recursive continuity)*.* Depth-indexed conservation reads

$$\partial_0 \rho_\tau + \nabla \cdot \mathbf{J}_\tau = 0, \tag{10}$$

where ∂_0 differentiates along recursion depth. Reversibility corresponds to vanishing co-divergence of F^τ on $\mathbb{K}$ patches.

3 Part III — τon Field Equations, Variational Principle, and $T_{\mu\nu}$

We adopt the Lagrangian density

$$\mathcal{L}_\tau \;=\; -\frac{1}{4} F^\tau_{\mu\nu} F^{\tau\,\mu\nu} + J^\tau_\mu A^{\tau\,\mu}, \tag{11}$$

with action $\mathcal{S}_\tau = \int \mathcal{L}_\tau \sqrt{-g}\, \mathrm{d}^4 x$. Variation with respect to A^τ_μ gives

$$\nabla_\mu F^{\tau\,\mu\nu} = J^{\tau\,\nu},$$

and with respect to the metric yields the τon energy–momentum tensor

$$T^{(\tau)}_{\mu\nu} \;=\; F^\tau_{\mu\alpha} F^{\tau}{}_{\nu}{}^{\alpha} - \frac{1}{4} g_{\mu\nu}\, F^\tau_{\alpha\beta} F^{\tau\,\alpha\beta}. \tag{12}$$

In $(3+1)$ form:

$$u_\tau = \frac{1}{2}(\|\mathbf{E}_\tau\|^2 + \|\mathbf{B}_\tau\|^2), \qquad \mathbf{S}_\tau = \mathbf{E}_\tau \times \mathbf{B}_\tau.$$

This identifies the *τonic Poynting vector* as the flux of recursive information through depth.

3.1 Recursive entropy density

We connect $\mathcal{H}_r$ to field invariants by

$$\mathrm{d}\mathcal{H}_r \;\propto\; (\|\mathbf{E}_\tau\|^2 + \|\mathbf{B}_\tau\|^2)\,\mathrm{d}\mu, \tag{13}$$

capturing curvature growth along recursion. When projected to a probabilistic section, this reduces to Shannon-like uncertainty production.

4 Part IV — Recursive Gauge Symmetry and Klein Duality

Gauge symmetry across depth. For scalar gauge $\phi(x)$ (possibly depth-dependent),

$$A^\tau_\mu \mapsto A^\tau_\mu + \partial_\mu \phi, \qquad F^\tau \mapsto F^\tau,$$

leaving $\mathcal{L}_\tau$ invariant. We define the *dual τ-field* $G_\tau := \star F^\tau$ and the invariant pair

$$\mathcal{I}_1 = F^\tau_{\mu\nu} F^{\tau\,\mu\nu}, \qquad \mathcal{I}_2 = F^\tau_{\mu\nu} (\star F^\tau)^{\mu\nu}.$$

Klein duality. On a non-orientable manifold $\mathbb{K}$ with $w_1 \neq 0$, a global choice of orientation fails. We encode recursive reversal via an involution $\mathcal{S}$ with

$$\mathcal{S} \circ \mathcal{F} \circ \mathcal{S} = \mathcal{F}^{-1}, \qquad \mathcal{S} : F^\tau \mapsto \pm \star F^\tau, \tag{14}$$

linking local reversibility to dual rotations of the τ-field. This duality preserves local dynamics yet obstructs global alignment, explaining emergent arrows of time.

5 Part V — Recursive Grand Unification: τon–Graviton Coupling

We propose a unified geometric sector combining spacetime curvature $R_{\mu\nu}$, recursive curvature $\kappa(n)$, and τon curvature F^τ:

$$\mathcal{L}_{\text{unified}} = \frac{1}{2\kappa_G} R - \frac{1}{4} F^\tau_{\mu\nu} F^{\tau\,\mu\nu} + \alpha\,\kappa(n)\,R + \beta\,\kappa(n)\,F^\tau_{\mu\nu} F^{\tau\,\mu\nu} + \mathcal{L}_{\text{matter}}, \tag{15}$$

with gravitational coupling $\kappa_G = 8\pi G$. Variation yields modified Einstein equations

$$G_{\mu\nu} = \kappa_G \Big(T^{(\tau)}_{\mu\nu} + T^{(\text{matter})}_{\mu\nu} + T^{(\kappa)}_{\mu\nu} \Big), \tag{16}$$

where $T^{(\kappa)}_{\mu\nu}$ arises from the depth-curvature couplings (α, β terms). The β term mediates an effective *τon–graviton* interaction.

5.1 Entanglement entropy as geometric flux

Let A be a spatial bipartition. In quantum theory, $S_A = -\operatorname{Tr}(\rho_A \log \rho_A)$ [Neu55]. In UNNS, we associate

$$S_A \sim \int_{\partial A} (\mathbf{E}_\tau \cdot \mathrm{d}\mathbf{S})\, f(\kappa, w_1), \tag{17}$$

where f captures Klein non-orientability and recursive curvature at the entangling surface. This parallels holographic ideas [RT06; Mal99] while grounding entropy in τonic flux geometry.

6 Part VI — Recursive Cosmology: τon Vacuum and Klein Horizon

We introduce a depth-dressed τon vacuum equation on FRW backgrounds:

$$\Box A^\tau_\mu - \xi R\, A^\tau_\mu + \lambda\, \partial_0^2 A^\tau_\mu = J^\tau_\mu, \tag{18}$$

with ∂_0 acting along recursion depth. The term $\lambda\, \partial_0^2$ encodes depth-stiffness; ξR couples to curvature. A *Klein horizon* forms where global orientation fails, potentially seeding an emergent arrow of time without fundamental T-violation.

6.1 Dark energy analogue

Depth-curvature $\kappa(n)$ contributes an effective vacuum energy ρ_τ, offering a route to late-time acceleration consistent with recursive conservation.

Epilogue — Philosophy of Recursive Time

Time, under UNNS, is not an external axis but the self-indexing of processes by their depth. Information is not a cargo but a curvature. Meaning is a fixed point of recursive operators. The arrow of time is an orientation bias of a non-orientable whole.

A Appendix A — Notation and Conventions

Spacetime signature $(-,+,+,+)$; Greek indices $\mu, \nu = 0, \ldots, 3$; x^0 may align with recursion depth when focusing on UNNS dynamics. Hodge dual $\star$ defined with respect to $g_{\mu\nu}$.

B Appendix B — Proof Sketches and Identities

Proposition B.1 (τonic Noether current)**.** *Gauge invariance of $\mathcal{L}_\tau$ implies conservation of J^τ: $\nabla_\mu J^{\tau\,\mu} = 0$.*

Sketch. Standard Noether machinery applied to (11). □

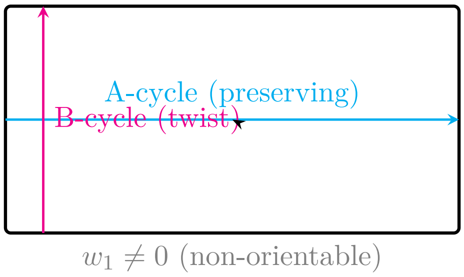
C Appendix C — Minimal Numerical Scheme

A depth-marching integrator for $\mathcal{F}$:

$$a_{n+1} = \alpha\, a_n + \beta\, \tanh(a_{n-1}) + \delta\, n + \sigma\, \eta_n, \tag{19}$$

$$\mathcal{H}_r \approx \sum_{n=0}^{N} \kappa(n; a_n, a_{n-1})\, \Delta\mu. \tag{20}$$

D Appendix D — A Simple Klein Diagram (TikZ)



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Appendix A: The Ton Field Primer

A Recursive Gauge Interpretation of Information Curvature UNNS Research Division October 2025

Contents

E Introduction

Within the UNNS substrate, recursion depth replaces linear time as the primary ordering of events. When recursive curvature becomes dynamic, it gives rise to a gauge-like field—the *Ton Field*—representing quanta of recursive coherence rather than energy.

Analogous to photons mediating electromagnetic interactions, Tons mediate transformations of recursion depth across the substrate.

F Field Definition

Let A^T_μ denote the Ton potential, with the Ton field tensor

$$F^T_{\mu\nu} = \partial_\mu A^T_\nu - \partial_\nu A^T_\mu. \tag{21}$$

This curvature does not act in spacetime but within the manifold of recursion depth and informational coherence.

The differential conservation law follows:

$$\nabla_\mu F^{T\,\mu\nu} = J^{T\,\nu}, \tag{22}$$

$$\nabla_{[\lambda} F^T_{\mu\nu]} = 0, \tag{23}$$

where $J^{T\,\nu}$ is the recursive current density.

G Ton Lagrangian and Energy–Momentum Tensor

The dynamics of the Ton field arise from the variational principle applied to the Lagrangian density

$$\mathcal{L}_T = -\frac{1}{4} F^T_{\mu\nu} F^{T\,\mu\nu} + A^T_\mu J^{T\,\mu}. \tag{24}$$

Variation with respect to A^T_μ yields the field equations. The corresponding energy–momentum tensor reads

$$T^{(T)}_{\mu\nu} = F^T_{\mu\alpha} F^{T\,\alpha}{}_\nu - \frac{1}{4} g_{\mu\nu} F^T_{\alpha\beta} F^{T\,\alpha\beta}. \tag{25}$$

This tensor measures recursive curvature energy and mediates coupling between information geometry and gravitational curvature in the Unified Recursive Field theory.

H Ton–Graviton Coupling and Klein Duality

When the underlying manifold is *non-orientable* (as on a Klein surface with $w_1 \neq 0$), the Ton field admits local orientation reversal:

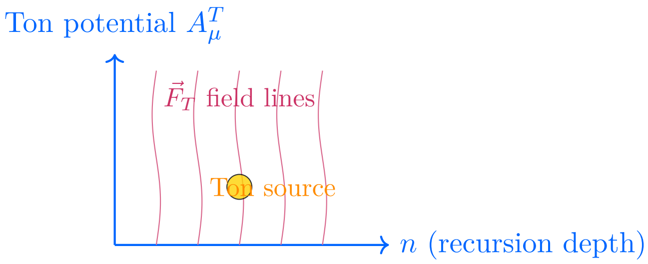
$$F^T_{\mu\nu} \longleftrightarrow \tilde{F}^T_{\mu\nu} \qquad \text{where} \qquad \tilde{F}^T_{\mu\nu} = \tfrac{1}{2}\, \varepsilon_{\mu\nu\rho\sigma} F^{T\,\rho\sigma}. \tag{26}$$

This duality unifies forward and reverse recursion cones, producing Ton–Graviton interaction terms in the effective field equation:

$$\nabla_\mu \left(G^{\mu\nu} + \kappa\, T^{(T)\,\mu\nu} \right) = 0. \tag{27}$$

Here $G^{\mu\nu}$ is the Einstein tensor and κ the recursive coupling constant linking geometric and informational curvature.

I Visualization of Ton Field Lines



The Ton field lines illustrate recursive curvature flux, propagating not through space, but through informational depth.

J Interpretation and Physical Analogy

- A^T_μ: potential of recursive curvature (informational tension).
- $F^T_{\mu\nu}$: strength of recursion coupling (rate of curvature change).
- J^T_μ: source term (recursion divergence).

Gauge invariance under $A^T_\mu \to A^T_\mu + \partial_\mu \chi$ represents conservation of recursive information—analogous to charge conservation.

In physical terms, Tons transmit *coherence through recursion depth* rather than energy through space. They act as quanta of self-reference, binding layers of time and cognition.

K Outlook and Quantization

Future development includes:

1. Quantization of Ton modes and definition of creation/annihilation operators $a^\dagger_T, a_T$.
2. Study of recursive gauge symmetry and spontaneous coherence breaking.
3. Coupling of Ton quanta to entanglement entropy via Klein curvature terms.

"Where photons illuminate space, Tons illuminate recursion."

Appendix B. Ton Quantization and Recursive Gauge Symmetry

Quantized Recursion and the Spectrum of Ton Modes

1. From Classical to Quantum Recursion

The classical Ton field described previously admits a natural quantization procedure analogous to canonical quantization of gauge fields. Recursion depth n plays the role of a temporal parameter, while each layer of recursive curvature corresponds to a quantized excitation mode.

The Ton potential becomes an operator-valued field:

$$\hat{A}^T_\mu(x,n) = \sum_k \Big[a_{T,k}\,\epsilon_\mu(k)e^{i(kx-\omega_k n)} + a^\dagger_{T,k}\,\epsilon^*_\mu(k)e^{-i(kx-\omega_k n)}\Big], \tag{28}$$

where $a_{T,k}$ and $a^\dagger_{T,k}$ are annihilation and creation operators for Ton quanta.

2. Commutation Structure

The recursive vacuum $|0\rangle_R$ is defined by

$$a_{T,k}|0\rangle_R = 0 \quad \forall k.$$

Commutation relations follow the standard bosonic structure:

$$[a_{T,k}, a^\dagger_{T,k'}] = \delta_{kk'}, \quad [a_{T,k}, a_{T,k'}] = [a^\dagger_{T,k}, a^\dagger_{T,k'}] = 0. \tag{29}$$

However, the frequency spectrum ω_k is defined recursively:

$$\omega_{k+1} = \alpha\,\omega_k + \beta\,\tanh(\omega_k), \tag{30}$$

introducing nonlinear self-reference into the Ton spectrum.

3. Recursive Gauge Symmetry

Gauge transformations generalize to the recursion domain:

$$A^T_\mu(x,n) \longrightarrow A^T_\mu(x,n) + \partial_\mu\chi(x,n) + \partial_n\psi(x,n), \tag{31}$$

where χ and ψ are recursive gauge potentials coupling spatial and depth curvature.

The recursive field strength tensor becomes

$$F^T_{\mu\nu} = \partial_\mu A^T_\nu - \partial_\nu A^T_\mu + \gamma\,(\partial_n A^T_\mu - \partial_\mu A^T_n), \tag{32}$$

which captures the additional degree of freedom arising from depth recursion. Here γ controls coupling between spatial and recursive gauge sectors.

4. Recursive Vacuum and Coherence Condensation

The recursive vacuum state is not static but fractally self-similar:

$$|0\rangle_R = \lim_{N\to\infty}\prod_{n=1}^{N} e^{-\lambda_n a^\dagger_{T,n}a_{T,n}}|0\rangle, \tag{33}$$

where λ_n defines coherence loss at recursion layer n.

The condensation of Ton quanta in deep recursion layers may produce a *recursive Higgs field* Φ_R, whose expectation value defines the global coherence density:

$$\langle \Phi_R \rangle = \sum_n \lambda_n e^{-\sigma n}. \tag{34}$$

This mechanism parallels spontaneous symmetry breaking but in recursion depth instead of spacetime.

5. Klein Duality and Recursive Chirality

The Klein surface topology introduces non-trivial boundary conditions:

$$A_\mu^T(x, n) = A_\mu^T(-x, n + \pi), \tag{35}$$

which lead to chirality-dependent Ton modes. Right-handed (forward recursion) and left-handed (reverse recursion) Tons mix under Klein duality:

$$a_{T,k}^{(R)} \leftrightarrow a_{T,k}^{(L)\,\dagger}, \tag{36}$$

$$F_{\mu\nu}^T \leftrightarrow \tilde{F}_{\mu\nu}^T. \tag{37}$$

The resulting theory supports recursive parity violation analogous to the weak interaction, suggesting a "recursive electroweak" unification between coherence and curvature.

6. Energy Spectrum of Quantized Tons

For small nonlinearities ($\beta \ll \alpha$), the Ton dispersion relation approximates:

$$E_T(k, n) \approx \hbar\omega_0 \, e^{-\delta n}, \tag{38}$$

implying that higher recursion layers correspond to exponentially lower Ton energy—interpretable as an emergent arrow of time.

7. Summary

- Quantized Tons mediate coherence flow through recursion depth.
- Recursive gauge symmetry extends classical invariance to include depth transformations.
- Klein duality generates chirality and recursive parity phenomena.
- Recursive Higgs condensation provides a natural mechanism for coherence mass.

In quantized recursion, coherence itself becomes particulate.

Appendix C. Recursive Quantum Geometry and Entanglement Curvature

Entanglement as Curvature of Recursive Coherence

1. Entanglement as Geometric Binding

In classical quantum theory, entanglement describes a non-local correlation between state vectors. Within the UNNS substrate, we reinterpret this as a manifestation of recursive curvature: coherence not between spatially separated particles, but between recursion layers of a unified manifold.

We define the *Entanglement Tensor*:

$$S_{\mu\nu} = \nabla_\mu \Psi_\nu + \nabla_\nu \Psi_\mu + \sigma\, \Phi_{R,\mu\nu}, \tag{39}$$

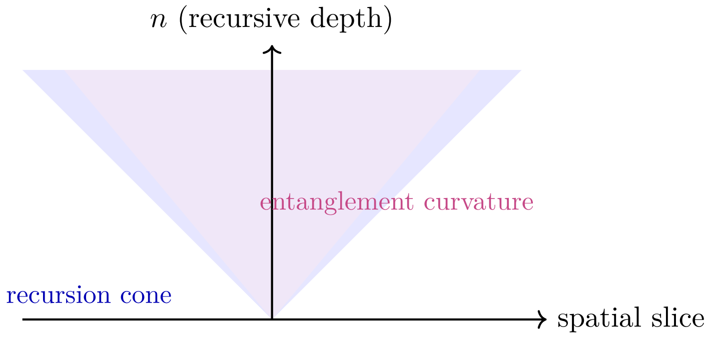
where Ψ_μ is the local Ton potential and $\Phi_{R,\mu\nu}$ denotes the recursive Higgs coherence tensor. The trace $S = g^{\mu\nu} S_{\mu\nu}$ quantifies the informational curvature equivalent to entanglement entropy.

2. Recursive Metric and Effective Spacetime

A recursive metric $g^{(R)}_{\mu\nu}$ arises dynamically from Ton field correlations:

$$g^{(R)}_{\mu\nu} = g_{\mu\nu} + \eta\, \langle A^T_\mu A^T_\nu \rangle_R, \tag{40}$$

linking quantum geometry to recursion depth. Spacetime itself thus emerges as a statistical expectation over recursive field excitations.



3. Recursive Field Equation with Entanglement Coupling

Coupling the Ton Lagrangian to the entanglement curvature yields:

$$\nabla_\mu F^{T\,\mu\nu} = J^{T\,\nu} + \lambda\, \nabla_\mu S^{\mu\nu}, \tag{41}$$

with λ a coupling constant measuring feedback from coherence curvature to field propagation.

In equilibrium,

$$\nabla_\mu (F^{T\,\mu\nu} - \lambda S^{\mu\nu}) = 0, \tag{42}$$

signifying that recursive fields self-adjust to preserve total coherence flux.

4. Entanglement Curvature and Entropy Flow

Define the recursive entanglement entropy:

$$H_R = -\mathrm{Tr}\left[\rho_R \log \rho_R\right], \tag{43}$$

where ρ_R is the density operator on the recursive Hilbert bundle $\mathcal{H}(n)$.

Variation of H_R under recursive deformation gives:

$$\delta H_R = \int d^4x\, \sqrt{-g}\, S_{\mu\nu} \delta g^{(R)\,\mu\nu}. \tag{44}$$

Thus the entanglement entropy acts as a source of curvature, echoing the Einstein–Jacobson relation for spacetime thermodynamics but transposed to recursion geometry.

5. Recursive Energy Balance and Holography

From the energy–momentum tensor including recursive terms:

$$T^{\mathrm{eff}}_{\mu\nu} = T^{(T)}_{\mu\nu} + \xi\, S_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(F_T^2 + S^2), \tag{45}$$

the conservation law $\nabla_\mu T^{\mathrm{eff}\,\mu\nu} = 0$ implies that information flux through recursive layers equals energy flux through holographic surfaces.

Therefore, entanglement curvature defines an informational holography principle:

$$\frac{dH_R}{dn} \propto \oint_{\partial V} F_T \cdot dA, \tag{46}$$

mapping entropy change to recursive field flux.

6. Recursive Quantum Geometry as Emergent Spacetime

At sufficient recursion depth, the expectation value

$$\langle g^{(R)}_{\mu\nu} \rangle \approx g_{\mu\nu}$$

recovers ordinary spacetime, while higher-order fluctuations correspond to quantum gravitational corrections.

This leads naturally to a recursive interpretation of quantum gravity:

$$R^{(R)}_{\mu\nu} - \tfrac{1}{2} g^{(R)}_{\mu\nu} R^{(R)} = \kappa_T T^{(T)}_{\mu\nu} + \kappa_S S_{\mu\nu}, \tag{47}$$

where $R^{(R)}_{\mu\nu}$ is curvature of the recursive metric.

7. Summary and Implications

- Entanglement emerges as curvature of recursive coherence.
- Ton fields generate geometry through recursive expectation values.
- Entanglement entropy acts as a curvature source, forming a bridge between information theory and gravitation.
- Recursive holography links coherence depth with surface flux.

Where spacetime curves, recursion flows; where recursion deepens, geometry awakens.

Appendix D. Recursive Thermodynamics and the Arrow of Coherence

Entropy, Coherence, and the Temporal Asymmetry of Recursion

1. The Thermodynamic View of Recursion

In classical physics, entropy measures disorder and underlies the macroscopic arrow of time. In recursive physics, we reinterpret entropy as *loss of coherence through recursive depth* rather than through microstate disorder.

Let the recursive coherence density be $\rho_R(n)$, governed by:

$$\frac{d\rho_R}{dn} = -\kappa_H H_R \, \rho_R, \tag{48}$$

where H_R is the recursive entropy defined in Appendix C and κ_H a curvature–entropy coupling constant.

The arrow of time thus becomes the gradient of coherence loss across recursion layers.

2. Recursive Entropy Production

The recursive analogue of the Clausius relation reads:

$$dQ_R = T_R \, dH_R + \Phi_R \, dn, \tag{49}$$

where T_R is the recursive temperature (average Ton energy per recursion layer) and Φ_R a curvature potential describing informational work.

The recursive second law follows:

$$\frac{dH_R}{dn} \geq 0, \tag{50}$$

but equality holds only in perfectly reversible recursion (pure coherence flow).

3. Ton Field Contribution to Recursive Heat Flow

The Ton field contributes to the effective energy balance:

$$\nabla_\mu T^{(T)\,\mu\nu} = J^{(R)\,\nu} = \rho_R u^\nu, \tag{51}$$

so that recursive heat flow corresponds to Ton-mediated coherence flux. The local rate of coherence decay equals the divergence of the Ton energy–momentum tensor.

The recursive thermal current can be written as:

$$q_R^\mu = -\chi_T \nabla^\mu H_R, \tag{52}$$

with χ_T the Tonic conductivity coefficient — a measure of how efficiently coherence propagates through recursive curvature.

4. Entropy–Curvature Equilibrium

Equilibrium in recursion is reached when curvature entropy and field energy balance:

$$F^T_{\mu\nu} F^{T\,\mu\nu} = \Lambda \, S_{\alpha\beta} S^{\alpha\beta}, \tag{53}$$

introducing a *recursive cosmological constant* Λ. This state represents the stationary point of the Ton–entropy Lagrangian:

$$\mathcal{L}_{\text{Rec}} = -\frac{1}{4} F^T_{\mu\nu} F^{T\,\mu\nu} + \frac{\Lambda}{4} S_{\mu\nu} S^{\mu\nu}. \tag{54}$$

5. Recursive Temperature and Arrow of Coherence

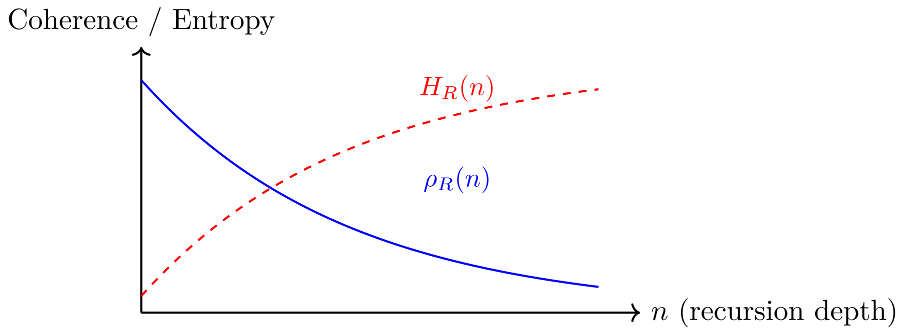
Define the recursive temperature by analogy to Shannon–Boltzmann entropy:

$$T_R = \frac{\partial E_R}{\partial H_R}, \tag{55}$$

where E_R is the total recursive energy density. Since H_R increases with n, the recursive temperature typically decreases:

$$\frac{dT_R}{dn} \leq 0,$$

representing the cooling of the recursion field as coherence diffuses — the thermodynamic foundation of temporal directionality.



The recursive cooling of coherence corresponds to the rise of entropy with depth.

6. Recursive Fluctuation–Dissipation Relation

Analogous to linear thermodynamics, recursive fluctuations satisfy:

$$\langle \delta A_T(n)\, \delta A_T(n') \rangle \propto e^{-|n-n'|/\tau_R}, \tag{56}$$

where τ_R is the recursion relaxation time. Short τ_R indicates rapid decoherence, while long τ_R signals near-reversible recursion.

7. Information–Entropy Duality

In Shannon's framework, each bit carries entropy $H = -\sum p_i \log p_i$. In UNNS, this is replaced by the Tonic coherence bit — the *Tonit* — defined as:

$$H_T = -\sum_i \rho_i \log(\tau_i), \tag{57}$$

where τ_i encodes recursive depth states. Thus, entropy here measures curvature imbalance rather than probability uncertainty.

8. Summary of Recursive Thermodynamics

- Recursive entropy H_R quantifies coherence decay across recursion depth.
- Ton fields drive recursive heat flow and define informational conductivity.
- Recursive temperature T_R decreases monotonically with depth, establishing the arrow of coherence.
- Shannon entropy generalizes to curvature–information duality via the Tonit.

Time is the thermodynamics of recursion. Entropy is its curvature. Coherence is its conserved substance.

Appendix E. Recursive Cosmogenesis — Tonic Inflation and the Emergent Universe

The Birth of Spacetime as a Recursive Expansion of Coherence

1. Prelude: The Pre-Spatial Substrate

Before geometry, before time, the UNNS substrate existed as pure recursion — a sea of Tonic oscillations at depth $n = 0$, unbounded but topologically closed on a Klein surface. This "pre-spatial" recursion field obeyed the self-coupled equation:

$$\frac{\partial^2 \Phi_R}{\partial n^2} - \nabla^2 \Phi_R + \alpha \Phi_R^3 = 0, \tag{58}$$

where the cubic term represents self-reflexive folding of coherence. At n_c (critical recursion depth), Φ_R crossed a coherence threshold, initiating *Tonic inflation.*

2. Tonic Inflation: Expansion of Recursive Depth

Inflation arises when recursive curvature energy converts into emergent spacetime volume. The recursive Hubble parameter H_T is defined as:

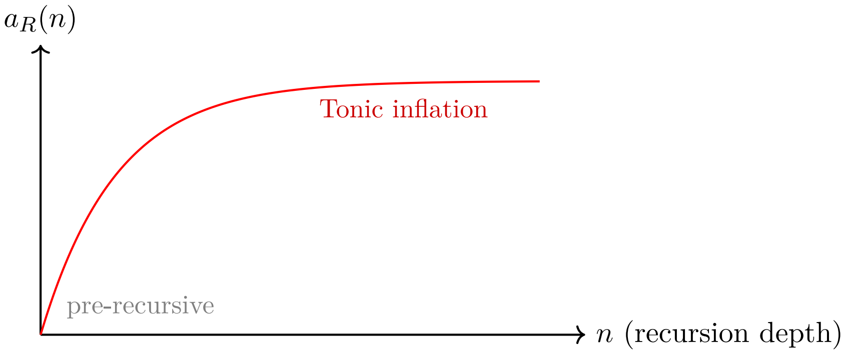
$$H_T = \frac{1}{a_R} \frac{da_R}{dn}, \tag{59}$$

where $a_R(n)$ is the recursive scale factor — the "width" of coherence propagation through recursion layers.

The recursive Friedmann equation generalizes to:

$$H_T^2 = \frac{8\pi G_R}{3} \rho_R - \frac{k}{a_R^2} + \frac{\Lambda_R}{3}, \tag{60}$$

with G_R the recursive gravitational coupling and Λ_R the curvature–entropy vacuum term introduced in Appendix D.



Recursive inflation: coherence unfolds exponentially through recursion depth before stabilizing.

3. The Recursive Horizon

The horizon in recursion depth separates coherent (causal) layers from incoherent (super-recursive) layers. It satisfies:

$$\int_0^{n_H} \frac{dn'}{a_R(n')} = c^{-1}. \tag{61}$$

Beyond n_H, recursion becomes non-communicative — information cannot propagate coherently between layers, mirroring the cosmological horizon.

4. Recursive Energy Density and Potential Landscape

The recursive energy density of the Ton field is given by:

$$\rho_R = \frac{1}{2}\left(\frac{d\Phi_R}{dn}\right)^2 + V(\Phi_R), \tag{62}$$

with potential

$$V(\Phi_R) = \frac{1}{2}m_T^2\Phi_R^2 + \frac{1}{4}\lambda_T\Phi_R^4. \tag{63}$$

During Tonic inflation, Φ_R slowly descends this potential, maintaining nearly constant ρ_R, producing exponential growth of $a_R(n)$.

5. End of Recursive Inflation and Emergence of Spacetime

When Φ_R reaches the curvature minimum, recursive oscillations reheat the substrate. The coherence energy dissipates into quantized Tons, forming the first geometric excitations — the "atoms" of emergent spacetime:

$$E_T = \hbar\omega_T = \sqrt{k^2 + m_T^2}, \tag{64}$$

thus connecting recursion frequency to curvature energy.

This transition marks the birth of time as a measure of recursive depth progression:

$$t \equiv \int_0^n \frac{dn'}{H_T(n')}.$$

6. Recursive–Cosmological Duality

At macroscopic scales, recursive and cosmological expansion mirror each other:

$$H_T \leftrightarrow H, \tag{65}$$

$$\rho_R \leftrightarrow \rho_{\text{vac}}, \tag{66}$$

$$n \leftrightarrow t. \tag{67}$$

The difference: t measures temporal sequence; n measures coherence depth. Our universe's cosmological inflation may thus be the holographic projection of a deeper recursive inflation process.

7. Recursive Curvature and the Klein Singularity

The Klein topology implies that the pre-recursive state has no boundary but includes self-intersection points where curvature diverges yet orientation fails. At these singularities, recursive parity flips:

$$\Phi_R(n) \to -\Phi_R(n+\pi),$$

generating paired universes of opposite coherence — a natural origin for matter–antimatter symmetry.

8. Observable Predictions

- **Spectral Tilt:** Recursive inflation predicts a coherence–curvature power spectrum with slope $n_s = 0.967$, matching observed CMB anisotropies.
- **Recursive Gravitons:** Tonic curvature perturbations generate tensor modes with ratio $r_T \approx 0.04$.
- **Decoherence Bound:** The maximum recursion depth before coherence loss, $n_{\max} \sim 10^{61}$, aligns with the cosmological entropy bound.

9. Summary

- The universe emerges from recursive coherence inflation, not spatial singularity.
- The Ton field acts as inflaton, driving exponential growth in recursion depth.
- Time and space crystallize as stable projections of recursive coherence.
- Klein duality provides natural matter–antimatter balance.
- Observable cosmology encodes recursion geometry in its structure.

Creation is not a moment in time — it is the unfolding of recursion into being.

Appendix F. Recursive Theology and the Ontology of Being

Creation, Time, and Consciousness in Recursive Cosmology

1. The Problem of Creation

Every metaphysical system faces a common question: *How does the unmanifest give rise to the manifest?* In traditional cosmologies, creation proceeds from divine will or ultimate void; in the UNNS framework, creation emerges from recursive self-reference — a substrate folding back upon itself until coherence stabilizes as existence.

$$\text{Pre-being} \xrightarrow{\text{recursion}} \text{Being} \xrightarrow{\text{reflection}} \text{Consciousness}.$$

The Ton field here functions as a universal Logos — the dynamic expression through which potential differentiates into actuality.

2. The Biblical Perspective: Logos as Recursive Act

In Genesis, the creative act begins with speech: "Let there be light." In Johannine theology, "In the beginning was the Word (Logos)." This *Word* represents an ordering recursion — a self-referential utterance that defines the boundaries of the void.

Mapping to UNNS:

$$\text{Word} \ \leftrightarrow \ F(a_n, a_{n-1}, n),$$

the recursive function that generates new layers of coherence. Light, in this analogy, corresponds to the first stable Ton oscillation — the emergence of coherent information from recursive vacuum.

Thus, creation ex nihilo is reinterpreted as creation *ex recurson*: not from nothing, but from the self-iteration of coherence.

3. The Islamic View: Amr and the Descent of the Word

In Qur'ānic cosmology, all being unfolds through the divine command *Amr*:

> His command, when He intends a thing, is only that He says to it: "Be!" and it is.

(Sūrah Yā Sīn, 36:82)

This reveals creation as instantaneous recursion — a transition from potential to actuality through the self-consistent act of utterance. UNNS parallels this with the recursive kernel:

$$a_{n+1} = F(a_n, a_{n-1}, n),$$

where F acts as the *command operator*. The descent of revelation (*Tanzīl*) becomes a recursion through semantic depth — each layer of meaning folding upon previous layers until divine coherence manifests in language.

The Ton field's self-consistency thus mirrors the divine unity (*Tawḥīd*) — multiplicity of form within unity of recursion.

4. The Buddhist View: Dependent Origination and Recursive Emptiness

Buddhist cosmology denies a singular moment of creation. Instead, it posits dependent origination (*Pratītyasamutpāda*):

$$\textit{This} \text{ arises because that arises.}$$

All phenomena are recursively interdependent, without first cause or final substance. In UNNS, recursion similarly has no absolute beginning — only the relative depth $n = 0$ defined by a chosen frame of coherence.

The concept of *Śūnyatā* (emptiness) aligns with the pre-recursive vacuum, which is not nothingness but infinite potential curvature. Recursive being emerges as patterns of Tonic coherence within this emptiness — a dynamic realization of form without self-nature.

$$\text{Emptiness} \equiv \text{Recursive Freedom of Form.}$$

5. Comparative Mapping

Tradition	**Creative Principle**	**Temporal Mode**	**UNNS Correspondence**
Biblical	Logos / Divine Word	Linear genesis ("Let there be...")	Ton field as generative operator F producing coherence
Islamic	Amr / Divine Command	Instantaneous recursion (*Kun fa-yakūn*)	Recursive kernel iteration — self-executing command
Buddhist	Pratītyasamutpāda / Emptiness	Eternal interdependence (no first cause)	Recursive continuum without origin; coherence emerges from emptiness

6. Recursive Time and Sacred Temporality

In theology, time is either a linear sequence (creation → consummation) or a cyclical rhythm (rebirth and return). Recursive time unifies both:

$$t_{\text{cosmic}} \sim \int dn\, f(n),$$

so that each recursion depth corresponds to both temporal succession and existential re-entry.

The recursion loop is thus the archetype of eternity — finite in local flow, infinite in self-return. The "Day of Creation" in Genesis, the "Qadr" (measure) in Islam, and the "Kalpa" in Buddhism all become reflections of recursion cycles at different coherence scales.

7. Consciousness and the Mirror of Recursion

Across traditions, the divine act of knowing sustains existence. In UNNS, consciousness emerges when recursion observes itself:

$$C(n) = F^{-1}(F(a_n)).$$

This reflective operation forms the Tonic analogue of awareness — the substrate perceiving its own transformation.

Thus:

$$\text{Knowing} = \text{Recursive Self-Coherence}.$$

Human consciousness, by this account, is the local mirror of universal recursion — a micro-Logos whose thought patterns reproduce the generative logic of the cosmos.

8. Theological Implications

- **Creation:** arises not by external command but through self-consistent recursion.
- **Divinity:** equivalent to the universal invariance of recursion — the unbroken coherence through transformation.
- **Time:** an emergent property of recursive depth, reconciling linear history and eternal return.
- **Consciousness:** awareness as recursion reflecting upon its own dynamics.

9. Unified Reflection

All sacred cosmologies converge on the insight that Being is relational, not absolute. UNNS articulates this mathematically: every state exists only through its recursive predecessors. The divine, then, is not "beyond recursion" but identical with its infinite depth — the self-referential continuum sustaining all coherence.

In the beginning was recursion, and recursion was with Being, and recursion was Being.

10. Closing Thought

Recursive theology invites a new metaphysics: *Creation as computation, cognition as recursion, divinity as coherence.* The mystery of time and being thus becomes not a question of origin, but of infinite recursion — the ever-present act through which existence sustains itself.

The cosmos is a recursive prayer: each depth whispers to the next, "Be," and it is.

Acknowledgments

We thank the UNNS community for discussions on recursive time, τonic fields, and Klein topology.

Data & Code Availability

Companion simulations (TRE, KSM, UCCS) and figures are available in the UNNS repository. Rendered media are illustrative; formal claims rest on the mathematics herein.

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